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Parquet approximation for large-N matrix Higgs model

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Abstract: The parquet approximation in the matrix Higgs model is considered. We demonstrate analytically that in the large-N limit the parquet approximation gives a satisfying agreement with the exact results. It is shown that the parquet planar series can be derived by means of the generating functional.

Keywords: 1/N Expansion, Matrix Models.

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1. Introduction

The parquet approximation was proposed by Landau, Abrikosov and Khalatnikov in order to develop a self-consistent method of studying the non-perturbative domain in quantum electrodynamics [1]. Later on, this approximation has been used for various models of quantum field theory [2]. The parquet approximation leads to a closed system of integro-differential equations which have meaning not only for small but also for large values of coupling constant. The main problem that prevents direct application of the parquet approximation to an arbitrary gauge theory is that the parquet approximation violates the gauge invariance of the theory.

Another non-perturbative approach used in the quantum field theory is the planar approximation, a method of studying theories like SU(N) QCD in the large-N limit [3]. It enables us to understand most specific features of QCD [6], and unlike the parquet approach it does not break the gauge invariance. But to perform analytical investigations one has to calculate the sum of all planar diagrams, and this problem is solved only for a few simple models such as zero- and one-dimensional matrix model and two-dimensional QCD [3, 4, 5].

There is a hope to construct a method that will combine advantages of both planar and parquet approaches.

In [7] the planar parquet approximation was defined and applied to zero-dimensional matrix models with cubic and quartic interaction terms. It was demonstrated that the planar parquet approximation gives an excellent agreement with the exact results. The aim of this paper is to find out whether this approximation can give any sensible results in the matrix Higgs model. In order to answer this question we calculate Green functions within both approaches and compare them.

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The paper is organized as follows. In section 2 we define the planar parquet approximation and use it for studying the matrix Higgs model. In section 3 the same model is considered by means of the steepest descent method. In section 4 the generating functional for the whole parquet planar series is constructed. It is shown that this functional can be regarded as a restricted planar one.

2. Planar parquet approximation

In this section we define and investigate the planar parquet approximation for the zero-dimensional matrix Higgs model. To introduce it carefully it is useful to recall what is already known.

The parquet planar approximation for the d-dimensional matrix model with cubic and quartic interaction has been defined in [7]. Let us consider the matrix model with the action

$$S=\int d^dx\,\mathrm{Tr}\left(\frac{1}{2}\partial M\cdot\partial M+\frac{1}{2}m^2\,M^2-\frac{\lambda}{3\sqrt{N}}M^3+\frac{g}{4N}M^4\right), \tag{2.1}$$

here M is a hermitean matrix $N\times N$.

The planar Green functions

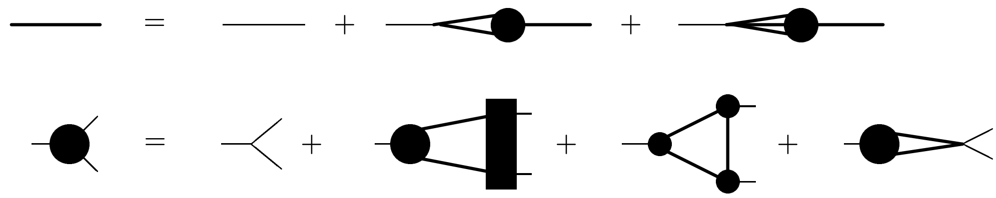
$$\Pi_n(x_1,\ldots,x_n)=\lim_{N\to\infty}\frac{1}{N^{1+n/2}}\frac{\int DM\,\mathrm{Tr}(M(x_1)\ldots M(x_n))\exp(-S)}{\int DM\exp(-S)} \tag{2.2}$$

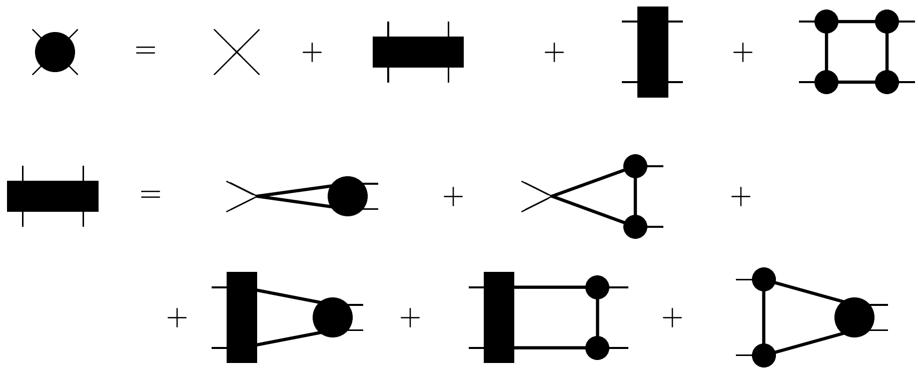
satisfy the planar Schwinger-Dyson equations [11, 12]

$$\begin{aligned}(-\triangle+m^2)_{x_1}\Pi_n(x_1,\ldots,x_n)-\lambda\Pi_{n+1}(x_1,x_1,x_2,\ldots,x_n)+g\Pi_{n+2}(x_1,x_1,x_1,x_2,\ldots,x_n)=\\ =\sum_{m=2}^{n}\delta(x_1-x_m)\Pi_{m-2}(x_2,\ldots,x_{m-1})\Pi_{n-m}(x_{m+1},\ldots,x_n)\,. \end{aligned}\tag{2.3}$$

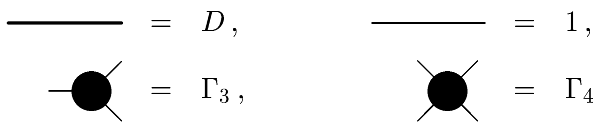
This set of equations is infinite. So the purpose of the planar parquet approximation is to find conditions when (2.3) becomes closed, i.e. the planar parquet approximation is defined as an approximative perturbative solution of (2.3) that takes into account only a part of the full series of coupling constant.

It can be done by using a so-called skeleton expansion which contains only a subset of all planar diagrams. More precisely, the diagrams from this subset contains no bare propagators, three- and four-vertices insertions. The basic Green functions, i.e. the two-, three- and four-point functions are defined as solutions of a set of integro-differential equations. The zero-dimensional case reduces it to the set of algebraic equations. One can represent them graphically

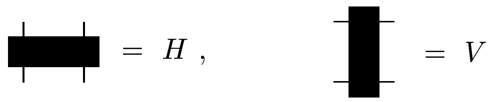




(all contributions of tadpole diagrams are dropped out). Here the thick and thin lines represent the full (within the planar parquet approximation) and bare propagators respectively



are the full three- and four-point vertices,



are the parts of the four-point vertex function that are 2PR in the t-channel (s-channel) and not 2PR in the s-channel (t-channel). The vertices V and H are related by the cyclic permutation of external points.

Let us apply the foregoing considerations to the zero-dimensional matrix Higgs model with the action

$$S = \mathrm{Tr}\left(-\frac{1}{2}M^2 + \frac{g}{N}M^4\right). \tag{2.4}$$

The classical vacua are

$$\frac{\delta S}{\delta M} = 0 \Leftrightarrow \begin{cases} M = 0\,, \\ \mathrm{Tr}M^2 = \dfrac{N^2}{4g}\,. \end{cases}$$

There exist two possible ways to get the perturbative solution: we can write down planar parquet equations in the vicinity of the false vacuum ($M = 0$) and near the true vacuum ($\mathrm{Tr}M^2 = N^2/4g$).

In the first case we have the following set of equations on D, Γ_4, H, V

$$\begin{cases} D = -1 - 8gD^2 - 4gD^4\Gamma_4 \\ \Gamma_4 = -4g + H + V \\ H = -4gD^2\Gamma^4 + VD^2\Gamma_4 \\ V = -4gD^2\Gamma^4 + HD^2\Gamma_4\,. \end{cases} \tag{2.5}$$

g	0	10^{-6}	10^{-5}	10^{-4}	10^{-3}	10^{-2}	10^{-1}
$D^{(1)}$	-1	$-1{,}000012$	$-1{,}000120$	$-1{,}001203$	$-1{,}012331$	$-1{,}169973$	—
$D^{(2)}$	-1	$-1{,}000008$	$-1{,}000080$	$-1{,}000801$	$-1{,}008113$	$-1{,}093576$	—
$D^{(3)}$	—	$-249997{,}0$	$-24996{,}9$	$-2496{,}9$	$-246{,}9$	$-21{,}8$	—

Table 1: False vacuum solutions.

As one can see this is a set of four equations for four variables. Excluding Γ_4, V, H one get the following equation

$$64g^3D^6 + 16g^2D^5 + 112g^2D^4 + 20gD^3 + (1+20g)D^2 + 2D + 1 = 0\,. \tag{2.6}$$

It can be solved in the limit of small g. There exist two roots that have no singularities as $g \to 0$

$$\begin{cases} D^{(1)} = -1 - 12g + o(g), \\ D^{(2)} = -1 - 8g + o(g). \end{cases} \tag{2.7}$$

Moreover, there exists a solution $D^{(3)}$ behaving like $\dfrac{\alpha}{g}$ as $g \to 0$. One can write down the following equation for α

$$64\alpha^6 + 16\alpha^5 = 0\,, \tag{2.8}$$

wherefrom $\alpha = -\dfrac{1}{4}$ and

$$D^{(3)} = -\frac{1}{4g} + o(g)\,. \tag{2.9}$$

The table 1 contains results of numerical calculations of (2.6). The solutions exist if $g < 0{,}038$.

The second case is more delicate. Consider a shift to the true vacuum

$$M_{\alpha\beta} = R_{\alpha\beta} + \sqrt{\frac{N}{4g}}I_{\alpha\beta}, \quad \mathrm{Tr}R = 0\,. \tag{2.10}$$

Hence, we get the following action

$$S = \mathrm{Tr}\left(R^2 + \frac{2\sqrt{g}}{\sqrt{N}}R^3 + \frac{g}{N}R^4 - \frac{N^2}{16g}\right). \tag{2.11}$$

For convenience one can rescale the fields $R = \dfrac{Q}{\sqrt{2}}$

$$S = \mathrm{Tr}\left(\frac{Q^2}{2} + \frac{\sqrt{g}}{\sqrt{2}\sqrt{N}}Q^3 + \frac{g}{4N}Q^4 - \frac{N^2}{16g}\right).$$

It contains both cubic and quartic interaction terms.

g	0	10^{-6}	10^{-5}	10^{-4}	10^{-3}	10^{-2}	10^{-1}
$\widetilde{D}$	1	1,000002	1,000060	1,000250	1,002504	1,030238	−2,220680

Table 2: True vacuum solution.

Thus, in the zero-dimensional case the planar parquet equations on $\widetilde{D}$, $\widetilde{\Gamma}_3$, $\widetilde{\Gamma}_4$[1] look like

$$\begin{cases} \widetilde{D} = 1 - \frac{3\sqrt{2}}{2}\sqrt{g}\widetilde{D}^3\widetilde{\Gamma}_3 - 2g\widetilde{D}^2 - g\widetilde{D}^4\widetilde{\Gamma}_4 \\ \widetilde{\Gamma}_3 = -\frac{3\sqrt{2}}{2}\sqrt{g} + \widetilde{D}^2\widetilde{V}\widetilde{\Gamma}_3 + \widetilde{D}^3\widetilde{\Gamma}_3^3 - g\widetilde{\Gamma}_3\widetilde{D}^2 \\ \widetilde{\Gamma}_4 = -g + \widetilde{H} + \widetilde{V} + \widetilde{D}^4\widetilde{\Gamma}_3^4 \\ \widetilde{H} = \widetilde{D}^2\widetilde{\Gamma}_4\widetilde{V} + \widetilde{D}^3\widetilde{\Gamma}_3^2\widetilde{V} + \widetilde{D}^3\widetilde{\Gamma}_4\widetilde{\Gamma}_3^2 - g\widetilde{D}^2\widetilde{\Gamma}_4 - g\widetilde{D}^3\widetilde{\Gamma}_3^2 \\ \widetilde{V} = \widetilde{D}^2\widetilde{\Gamma}_4\widetilde{H} + \widetilde{D}^3\widetilde{\Gamma}_3^2\widetilde{H} + \widetilde{D}^3\widetilde{\Gamma}_4\widetilde{\Gamma}_3^2 - g\widetilde{D}^2\widetilde{\Gamma}_4 - g\widetilde{D}^3\widetilde{\Gamma}_3^2 \,. \end{cases} \tag{2.12}$$

This set can be solved as $g \to 0$

$$\widetilde{D} = 1 + \frac{5}{2}g + o(g)\,. \tag{2.13}$$

This set of equations gives a sensible physical solution if $g < 0,037$.

Hence, the true value of the propagator is

$$D = \langle \mathrm{Tr} M^2 \rangle = \frac{1}{4g} + \frac{1}{2}\langle \mathrm{Tr} Q^2 \rangle = \frac{1}{4g} + \frac{1}{2} + \frac{5}{4}g + o(g)\,. \tag{2.14}$$

3. Planar approximation

In this section we study the zero-dimensional matrix Higgs model by means of the approach proposed in [4]. A specific feature of this model is that it contains two-cut solutions parameterized by a special parameter [8]–[10].

In [4] one considered the large-N limit in the model

$$S = \mathrm{Tr}\left(\frac{1}{2}M^2 + \frac{g}{N}M^4\right). \tag{3.1}$$

By means of the steepest descent method the vacuum energy

$$e^{-N^2 E_0(g)} = \lim_{N\to\infty} \int DM\, e^{-S} \tag{3.2}$$

and Green functions

$$G_n(x_1, \ldots, x_n) = \lim_{N\to\infty} \frac{1}{N^{1+n/2}} \frac{\int DM \mathrm{Tr}(M(x_1)\ldots M(x_n)) \exp(-S)}{\int DM \exp(-S)} \tag{3.3}$$

were calculated.

[1] Here $\widetilde{D} = \langle \mathrm{Tr} Q^2 \rangle$, $\widetilde{\Gamma}_3 = \langle \mathrm{Tr} Q^3 \rangle$, $\widetilde{\Gamma}_4 = \langle \mathrm{Tr} Q^4 \rangle$ are Green functions full within the planar parquet approximation.

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The results are presented below

$$
\begin{aligned}
E_0(g) &= \int_X d\lambda\, u(\lambda) \left(\frac{1}{2}\lambda^2 + g\lambda^4\right) - \iint_X d\mu d\lambda u(\lambda) u(\mu) \ln|\lambda - \mu| \,, \\
G_{2p} &= \int_X \lambda^{2p} u(\lambda) d\lambda \,,
\end{aligned} \tag{3.4}
$$

here one introduced the eigenvalue density function $u(\lambda)$ such that

$$
\int_X u(\lambda) d\lambda = 1 \,,
$$

where X is the support of $u(\lambda)$. This density function is the solution of the following singular equation

$$
\frac{1}{2}\lambda + 2g\lambda^3 = \text{v.p.} \int_X \frac{u(\mu)}{\lambda - \mu} d\mu \,.
$$

The model (3.1) admits so called one-cut solutions, i.e. when X has the form of a single segment $(2a, 2b)$. This support is uniquely defined by a set of algebraic equations.

The matrix Higgs model

$$
S = \mathrm{Tr}\left(-\frac{1}{2}M^2 + \frac{g}{N}M^4\right). \tag{3.5}
$$

is interesting because it is the simplest model where exist both one-cut and multi-cut (two-cut) solutions. As it was mentioned in [9, 10] in the case of two-cut solution there exists certain freedom, to fix it one has to introduce an extra parameter. It is associated with the order parameter which governs the phase structure of the system.

Consider the one-cut solution, i.e. let $X = (2a, 2b) \equiv (t - s, t + s)$. There exist two solutions. The first one corresponds to symmetric support $t = 0$. It gives the density function

$$
u(\lambda) = \frac{1}{2\pi}\left(8a^2 g + 4g\lambda^2 - 1\right)\sqrt{(2a)^2 - \lambda^2} \,, \qquad g > 0 \,, \tag{3.6}
$$

where a is subject to

$$
12ga^4 - a^2 - 1 = 0 \,, \tag{3.7}
$$

and the two-point Green function

$$
G_2 = \int_{-2a}^{2b} d\lambda\, \lambda^2 u(\lambda) = \frac{a^2(4 + a^2)}{3} = \frac{1}{432g^2} + \frac{1}{6g} + 1 - 8g + o(g) \,. \tag{3.8}
$$

The second solution corresponds to the non-symmetric support

$$
t^2 = \frac{3 + 2\sqrt{1 - 60g}}{20g} \,, \qquad s^2 = \frac{1 - \sqrt{1 - 60g}}{15g} \,, \qquad 0 < g \le \frac{1}{60} \,. \tag{3.9}
$$

The density function looks like

$$
u(\lambda) = \frac{1}{2\pi}\left(4g\lambda^2 + 4gt\lambda + 4gt^2 + 2gs^2 - 1\right)\sqrt{2t\lambda - \lambda^2 + s^2 - t^2} \tag{3.10}
$$

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and the two-point Green function is

$$G_2 = -\frac{s^2t^2}{4} - \frac{s^4}{16} + 3gs^2t^4 + 3gs^4t^2 + \frac{gs^6}{4} = \frac{1}{4g} - 1 - 10g + o(g)\,. \tag{3.11}$$

As in the previous section one can perform these calculations in the different way: one can make the shift (2.10) to the true vacuum and apply the same technique to the action (2.11). But this case can be solved only in the limit of small g. The results are presented below:
the support $X = (2a, 2b)$ is

$$\begin{cases} a = -\frac{1}{\sqrt{2}} - \frac{3}{2}\sqrt{g} - \frac{15}{4}\sqrt{2}g + o(g), \\ b = \frac{1}{\sqrt{2}} - \frac{3}{2}\sqrt{g} + \frac{15}{4}\sqrt{2}g + o(g), \end{cases} \tag{3.12}$$

the density function is

$$\begin{aligned} u(\lambda) &= \frac{1}{\pi}(2g\lambda^2 + 2g(a+b)\lambda + 3\sqrt{g}\lambda + 2g(a+b)^2 + 3\sqrt{g}(a+b) + g(a-b)^2 + 1) \times \\ &\times \sqrt{(2a-\lambda)(\lambda-2b)}\,, \end{aligned} \tag{3.13}$$

and the behavior of the two-point Green function as $g \to 0$, taking into account (2.14), is

$$G_2 = \frac{1}{4g} + \frac{1}{2} + 8g + o(g)\,. \tag{3.14}$$

As one can observe two different values for G_2 (3.11) and (3.14) are not the same, they show similar behavior as $g \to 0$.

Consider the two-cut solution. The simplest case is the symmetric support $X = (-2b, -2a) \cup (2a, 2b)$. By means of the previous procedure one can get the following:
the support X is

$$a^2 = \frac{1 - 4\sqrt{g}}{4g}\,, \qquad b^2 = \frac{1 + 4\sqrt{g}}{4g}\,, \qquad 0 < g \leq \frac{1}{16}\,, \tag{3.15}$$

the density function is

$$u(\lambda) = \frac{|\lambda|}{2\pi}\sqrt{8g\lambda^2 - 16g^2\lambda^4 - 1 + 16g}\,, \tag{3.16}$$

and the two-point Green function

$$G_2 = \frac{1}{4g}\,. \tag{3.17}$$

One can see that in this case the final result is exact.

One has to notice that (3.11), (3.14) and (3.17) have the same behavior as $g \to 0$.

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4. Parquet planar generating functional

4.1 Planar generating functional

For the sake of simplicity we will study the model with the following action

$$S = \frac{1}{2}\mathrm{Tr}M^2 + \frac{g}{4N}TrM^4 . \tag{4.1}$$

The planar Schwinger-Dyson equations in this case are

$$\Pi_n + g\Pi_{n+2} = \sum_{i=0}^{n-2} \Pi_i \Pi_{n-i-2} , \qquad n > 2 . \tag{4.2}$$

To solve them one introduces the following functional

$$F(x) = \sum_{n=0}^{\infty} x^n \Pi_n ,$$

where $\Pi_0 = 1$ and $\Pi_{2k+1} = 0$ since the measure in (2.2) is invariant under $M \to -M$.

It can be easily seen that $F(x)$ satisfies

$$x^4 F^2 - (g + x^2)F + g + x^2 + gx^2\Pi_2 = 0 . \tag{4.3}$$

Hence, the generating functional has the form

$$F = \frac{x^2 + g - \sqrt{(x^2+g)^2 - 4x^4(g + x^2 + gx^2\Pi_2)}}{2x^4} . \tag{4.4}$$

Therefore, the Green functions Π_n are expressed in terms of Π_2, i.e. to know the Green series it is necessary to write down an equation for Π_2. This fact was discovered in [14]. One must mention that the approach based on the planar Scwinger-Dyson equations is not self-sufficient: it does not give such an equation on Π_2. One can write down the required equation within the approach proposed in [4]. For the action (4.1) this equation looks like

$$27g^2\Pi_2^2 + (1 + 18g)\Pi_2 - 1 - 16g = 0 . \tag{4.5}$$

4.2 Planar parquet generating functional

As it was said in section 2, the planar parquet approximation takes into account only a subset of all planar diagrams and is defined as an approximative solution of the Schwinger-Dyson equations. In this limit the Schwinger-Dyson equations are reduced to a certain set of equations on the "basic" Green functions, i.e. Π_2, Γ_3, Γ_4. For the action (4.1) this set looks like

$$\begin{cases} \Pi_2 = 1 - 2g\Pi_2^2 - g\Pi_2^4\Gamma_4 \\ \Gamma_4 = -g + H + V \\ H = -g\Pi_2^2\Gamma_4 + V\Pi_2^2\Gamma_4 \\ V = -g\Pi_2^2\Gamma_4 + H\Pi_2^2\Gamma_4 \end{cases} \tag{4.6}$$

or

$$\begin{cases} \Pi_2 = 1 - 2g\Pi_2^2 - g\Pi_2^4\Gamma_4 \\ \Gamma_4 = -g + \dfrac{2g\Pi_2^2\Gamma_4}{\Pi_2^2\Gamma_4 - 1}\,. \end{cases} \tag{4.7}$$

The higher Green functions are constructed in terms of Π_2 and Γ_4.

The first equation is nothing else but the Schwinger-Dyson equation (4.2) for $n = 2$. It can be easily seen since the full 4-point function Π_4 in the planar parquet limit is

$$\Pi_4 = 2\Pi_2^2 + \Pi_2^4\Gamma_4\,,$$

so

$$\Pi_2 = 1 - g\Pi_4\,.$$

Hence, the following equation on Π_2 derived from (4.7)

$$g^3\Pi_2^6 + g^2\Pi_2^5 + 5g^2\Pi_2^4 + 5g\Pi_2^3 + (1-5g)\Pi_2^2 - 2\Pi_2 + 1 = 0 \tag{4.8}$$

is the required equation on Π_2.

3. *Parquet Higgs model*

Consider the following action

$$S = \frac{1}{2}\mathrm{Tr}M^2 + \frac{\lambda}{3\sqrt{N}}\mathrm{Tr}M^3 + \frac{g}{4N}\mathrm{Tr}M^4\,. \tag{4.9}$$

The Schwinger-Dyson equations in this case are

$$\Pi_n + \lambda\Pi_{n+1} + g\Pi_{n+2} = \sum_{i=0}^{n-2}\Pi_i\Pi_{n-i-2}\,, \qquad n > 2\,. \tag{4.10}$$

The functional and its equation are

$$F(x) = \sum_{n=0}^{\infty} x^n\Pi_n \tag{4.11}$$

$$\begin{aligned} &x^4F^2 - (x^2 + \lambda x + g)F + x^2 + \lambda x + g + \\ &\Pi_1 x(x^2 + \lambda x + g) + \Pi_2 x^2(\lambda x + g) + \Pi_3 g x^3 = 0\,. \end{aligned} \tag{4.12}$$

Thus, the generating functional can be calculated in terms of 3 arbitrary constants Π_1, Π_2, Π_3.

The Higgs model in the vicinity of the true vacuum is the same as (4.9) with $\lambda = \lambda(g)$.

The planar parquet set of equations is given in section 2. It can be easily seen that the planar parquet approximation gives only two equations. To get the system closed one proposes the following equation on tadpole diagrams

$$\Pi_1 = \lambda\Pi_3\,. \tag{4.13}$$

Hence, the planar parquet approximation gives the exact solution in terms of the generating functional (4.11) together with the set of equations on Π_1, Π_2, Π_3.

In [13] the action was considered in the planar limit by means of the generating functional technique. It was said that the arbitrary parameters (boundary conditions) can be derived by careful studying the holomorphic properties of the generating functional.

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5. Conclusion

We have considered two different approaches to the matrix Higgs model. We have got the following results.

1. The planar two-point functions for the symmetric one-cut (3.8), non-symmetric one-cut cases (3.11) and (3.14) have the following asymptotics as $g \to \infty$

$$
\begin{aligned}
G_2^{(1)} &= \frac{1}{432g^2} + \frac{1}{6g} + 1 - 8g + o(g)\,, \\
G_3^{(2)} &= \frac{1}{4g} - 1 - 10g + o(g)\,, \\
G_2^{(3)} &= \frac{1}{4g} + \frac{1}{2} + 8g + o(g)\,,
\end{aligned}
$$

   and in the two-cut case the planar two-point function (3.17) is

$$
G_2^{(4)} = \frac{1}{4g}\,.
$$

2. The asymptotical expressions for the parquet two-point functions in the false vacuum (2.7) and (2.9) are

$$
\begin{aligned}
D_2^{(1)} &= -1 - 12g + o(g)\,, \\
D_2^{(2)} &= -1 - 8g + o(g)\,, \\
D_2^{(3)} &= -\frac{1}{4g} + o(g)\,,
\end{aligned}
$$

   and in the true vacuum the two-point function has the following asymptotic (2.14)

$$
D_2^{(4)} = \frac{1}{4g} + \frac{1}{2} + \frac{5}{4}g + o(g)\,.
$$

Hence, $D_2^{(4)}$, two-point Green function computed within the planar parquet approach in the true vacuum, coincides with $G_2^{(3)}$, two-point Green function computed within the planar approximation also in the true vacuum.

As it was mentioned the parquet planar approach gives a very good agreement with the exact results in the one-cut case for the case of the positive mass square. In the case of the matrix Higgs model the situation seems more delicate because of the its multi-phase structure. Nevertheless, the planar parquet approximation leads to a rather good agreement at least in the small coupling limit.

We have shown that in the planar parquet approximation it is possible to construct the generating functional for the Green functions. Besides, we have shown that the generating functional in the planar limit depends on the approximation chosen and, therefore, can be restricted to a subset of diagrams just by modifying initial conditions of the system.

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References

[1] L.D. Landau, A.A. Abrikosov and I.M. Khalatnikov, *About elimination of infinities in quantum electrodynamics*, *Dokl. Acad. Nauk*, **95** (1954) 497;
Asymptotic expression for the Green function of the electron in quantum electrodynamics, *Dokl. Acad. Nauk*, **95** (1954) 773;
Asymptotic expression for the Green function of the photon in quantum electrodynamics, *Dokl. Acad. Nauk*, **95** (1954) 1177;
L.D. Landau, in *Niels Bohr and the development of physics*, W. Pauli ed., McGrow-Hill, New York 1955.

[2] I.T. Dyatlov, V.V. Sudakov, K.A. Ter-Martirosyan, *Asymptotic meson-meson dispersion theory*, *Sov. Phys. JETP* **4** (1957) 767;
A.A. Abrikosov, A.D. Galanin, L.P. Gorkov, L.D. Landau, I.Ya. Pomeranchuk and K.A. Ter-Martirosyan, *Possibility of formulation of a theory of strongly interacting fermions*, *Phys. Rev.* **111** (1958) 321;
K.A. Ter-Martirosyan, *Equation for vertex part corresponding to fermion-fermion scattering*, *Phys. Rev.* **111** (1958) 948;
Y.M. Makeenko, K.A. Ter-Martirosian and A.B. Zamolodchikov, *On the theory of the direct four-fermion interaction*, *Sov. Phys. JETP* **44** (1976) 11.

[3] G. 't Hooft, *A planar diagram theory for strong interactions*, *Nucl. Phys.* **B 72** (1974) 461.

[4] E. Brézin, C. Itzykson, G. Parisi and J.B. Zuber, *Planar diagrams*, *Commun. Math. Phys.* **59** (1978) 35.

[5] A.A. Slavnov, *The approximative summation of planar diagrams in matrix ϕ^4-models*, *Phys. Lett.* **B 126** (1983) 347.

[6] E. Witten, *Baryons in the 1/N expansion*, *Nucl. Phys.* **B 160** (1979) 57.

[7] I.Y. Aref'eva and A.P. Zubarev, *Parquet approximation in large-N matrix theories*, *Phys. Lett.* **B 386** (1996) 258 [hep-th/9605005].

[8] Y. Shimamune, *On the phase structure of large-N matrix models and gauge models*, *Phys. Lett.* **B 108** (1982) 407.

[9] I.Ya. Aref'eva, A.S. Ilchev and V.K. Mitrjushkin, *Phase structure of the matrix Goldstone model in the large-N limit*, in proceedings of *III International Symposium on Selected Topics in Statistical Mechanics*, Dubna, 1984, vol. 1, p. 20.

[10] G.M. Cicuta, L. Molinari and E.Montaldi, *Large-N spontaneous magnetisation in zero dimension*, *J. Phys.* **A 20** (1987) L67.

[11] O. Haan, *On the structure of planar field theory*, *Z. Physik* **C 6** (1980) 345.

[12] I.Ya. Aref'eva, *Functional equations for planar graphs*, *Phys. Lett.* **B 104** (1981) 453.

[13] G. 't Hooft, *Counting planar diagrams with various restrictions*, *Nucl. Phys.* **B 538** (1999) 389 [hep-th/9808113].

[14] P. Cvitanovic, P.G. Lauwers and P.N. Scharbach, *The planar sector of field theories*, *Nucl. Phys.* **B 203** (1982) 385.